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New sequence spaces defined by infinite matrix

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Abstract

In this paper, we introduce some new sequence spaces ideal convergence and infinite matrix in 2-normed space and we also present some relations related to these sequence spaces.

1 Introduction

Throughout, ω will stand for the set of all sequences of real numbers and by l_∞ and c, we denote the Banach spaces of bounded and convergent sequences $x = (x_k)$ normed by $\|x\| = \sup_n |x_n|$, respectively.

In summability theory, the concept of almost convergence was first introduced by Banach [1] as following: A linear functional L on l_∞ is said to be a Banach limit if it has the following properties:

(i) $L(x) \geq 0$ if $n \geq 0$ (i.e., $x_n \geq 0$ for all n),

(ii) $L(e) = 1$ where $e = (1, 1, \ldots)$,

(iii) $L(Dx) = L(x)$,

where the shift operator D is defined by $D(x_n) = \{x_{n+1}\}$. Let B be the set of all Banach limits on l_∞. A sequence $x \in \ell_\infty$ is said to be almost convergent

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if all Banach limits of x coincide. Let $\hat{c}$ denote the space of almost convergent sequences. Lorentz [12] has shown that

$$\hat{c} = \left\{ x \in l_\infty : \lim_m t_{m,n}(x) \text{ exists uniformly in } n \right\}$$

where

$$t_{m,n}(x) = \frac{x_n + x_{n+1} + x_{n+2} + \cdots + x_{n+m}}{m+1}.$$

Let X and Y be two nonempty subsets of the space w of complex sequences. Let $B = (b_{nk}), (n, k = 1, 2, ...)$ be an infinite matrix of complex numbers. We write $Bx = (B_n(x))$ if $B_n(x) = \sum_k b_{nk} x_k$ converges for each n (where $\sum_k$ denotes summation over k from $k = 1$ to $k = \infty$). If $x = (x_k) \in X \Rightarrow Bx = (B_n(x)) \in Y$ we say that B defines a (matrix) transformation from X to Y and we denote it by $B : X \to Y$.

P. Kostyrko et al. [9] introduced the notion of ideal convergence as a generalization of statistical convergence. Some more papers of ideals can be seen in ([2], [4], [10], [14], [15], [18] and [23]). On the other hand, Gähler [5] introduced the concept of 2-normed space as an interesting non-linear generalization of a normed linear space which was subsequently studied by many authors (see, [6], [7] and [17]). More details of this concept can be see in (see, [8], [19], [20] and [22]).

A family $\mathcal{I} \subset 2^Y$ of subsets a nonempty set Y is said to be an ideal in Y if

(i) $A, B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$;

(ii) $A \in \mathcal{I}$, $B \subset A$ imply $B \in \mathcal{I}$, while an admissible ideal $\mathcal{I}$ of Y further satisfies $\{x\} \in \mathcal{I}$ for each $x \in Y$.

Given $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal in $\mathbb{N}$. The sequence $(x_n)_{n \in \mathbb{N}}$ in X is said to be $\mathcal{I}-$convergent to $x \in X$, if for each $\varepsilon > 0$ the set $A(\varepsilon) = \{n \in \mathbb{N} : \|x_n - x\| \geq \varepsilon\}$ belongs to $\mathcal{I}$, (see, [9, 10]).

Let X be a real vector space of dimension d, where $2 \leq d < \infty$. A $2-$norm on X is a function $\|.,.\| : X \times X \to \mathbb{R}$ which satisfies;

(i) $\|x, y\| = 0$ if and only if x and y are linearly dependent;

(ii) $\|x, y\| = \|y, x\|$;

(iii) $\|\alpha x, y\| = |\alpha| \|x, y\|$, $\alpha \in \mathbb{R}$;

(iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$.

The pair $(X, \|.,.\|)$ is then called a $2-$normed space [6]. Also, recall in [11] that an Orlicz function $M : [0,\infty) \to [0,\infty)$ is continuous, convex, nondecreasing function such that $M(0) = 0$ and $M(x) > 0$ for $x > 0$, and $M(x) \to \infty$ as $x \to \infty$. Later on Orlicz function was used to define sequence spaces by Parashar and Choudhary [16] and others ([3], [19], [20], [21],[22], [24], [25]).

The goal of this paper is to introduce some new sequence spaces in 2-normed spaces by using Orlicz functions, infinite matrix and ideals.

2 Main results

Let $\mathcal{I}$ be an admissible ideal of $\mathbb{N}$, M be an Orlicz function, $(X, \|.,.\|)$ be a $2-$normed space and $B = (b_{n,k})$ be a nonnegative matrix. Also let $p = (p_k)$ be a bounded sequence of positive real numbers. By $S(2-X)$, we indicate the space of all sequences defined over $(X, \|.,.\|)$. Now we write the following sequence spaces: for each m,

$$\hat{W}^{\mathcal{I}}(B, M, p, \|.,.\|) =$$

$$\left\{ \begin{array}{c} x \in S(2-X) : \forall \varepsilon > 0 \ \left\{ n \in \mathbb{N} : \sum\limits_{k=1}^{\infty} b_{nk} \left[M\left(\left\| \frac{t_{km}(x-L)}{\rho}, z \right\| \right) \right]^{p_k} \geq \varepsilon \right\} \in \mathcal{I} \\ \text{for some } \rho > 0,\ L \in X \text{ and each } z \in X, \text{ uniformly in } m, \end{array} \right\},$$

$$\hat{W}_0^{\mathcal{I}}(B, M, p \|.,.\|) =$$

$$\left\{ \begin{array}{c} x \in S(2-X) : \forall \varepsilon > 0 \ \left\{ n \in \mathbb{N} : \sum\limits_{k=1}^{\infty} b_{nk} \left[M\left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} \geq \varepsilon \right\} \in \mathcal{I} \\ \text{for some } \rho > 0, \text{ and each } z \in X, \text{ uniformly in } m, \end{array} \right\},$$

$$\hat{W}_\infty(B, M, p, \|.,.\|) =$$

$$\left\{ \begin{array}{c} x \in S(2-X) : \exists K > 0 \text{ s.t. } \sup\limits_{n \in \mathbb{N}} \sum\limits_{k=1}^{\infty} b_{nk} \left[M\left(\left\| \dfrac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} \leq K \\ \text{for some } \rho > 0, \text{ and each } z \in X. \end{array} \right\},$$

$\hat{W}_\infty^{\mathcal{I}}(B, M, p, \|.,.\|) = \{x \in S(2-X) : \exists K > 0$, s.t.

$$\left\{ n \in \mathbb{N} : \sum_{k=1}^{\infty} b_{nk} \left[M\left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} \geq K \right\} \in \mathcal{I}$$

for some $\rho > 0$, and each $z \in X$, uniformly in m.

Let us consider a few special cases of the above sets .

(i) If $M(x) = x$, for all $x \in [0, \infty)$, then the above classes of sequences are denoted by $\hat{W}^{\mathcal{I}}(B, p, \|., .\|), \hat{W}_0^{\mathcal{I}}(B, p, \|., .\|), \hat{W}_\infty(B, p, \|., .\|)$ and $\hat{W}_\infty^{\mathcal{I}}(B, p, \|., .\|)$, respectively.

(ii) If $p_k = 1$ for all $k \in N$, then we denote the above classes of sequences by $\hat{W}^{\mathcal{I}}(B, M, \|., .\|), \hat{W}_0^{\mathcal{I}}(B, \|., .\|), \hat{W}_\infty(B, \|., .\|)$ and $\hat{W}_\infty^{\mathcal{I}}(B, \|., .\|)$, respectively.

(iii) If $M(x) = x$, for all $x \in [0, \infty)$, and $p_k = 1$ for all $k \in N$, then we denote the above spaces by $\hat{W}^{\mathcal{I}}(B, \|., .\|), \hat{W}_0^{\mathcal{I}}(B, \|., .\|), \hat{W}_\infty(B, \|., .\|)$ and $\hat{W}_\infty^{\mathcal{I}}(B, \|., .\|)$, respectively.

(iv) If we take $B = (b_{nk})$ as

$$b_{nk} = \begin{cases} \frac{1}{n}, & \text{if } n \geq k, \\ 0, ‘ & \text{otherwise} \end{cases}$$

then the above classes of sequences are denoted by $\hat{W}^{\mathcal{I}}(C, M, p, \|, ., \|)$, $\hat{W}_0^{\mathcal{I}}(C, M, p, \|., .\|), \hat{W}_\infty(C, M, p, \|., .\|), \hat{W}_\infty^{\mathcal{I}}(C, M, p, \|., .\|)$ respectively, which were defined and studied by Savas [20]

(v) If we take $B = (b_{nk})$ is a de la Valee poussin mean, i.e.,

$$b_{nk} = \begin{cases} \frac{1}{\lambda_n}, & \text{if } k \in I_n = [n - \lambda_n + 1, n], \\ 0, & \text{otherwise} \end{cases}$$

where (λ_n) is a non-decreasing sequence of positive numbers tending to ∞ and $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$, then the above classes of sequences are denoted by $\hat{W}^{I}(M, \lambda, p\|., .\|), \hat{W}_0^{\mathcal{I}}(M, \lambda, p\|., .\|), \hat{W}_\infty(M, \lambda, p\|., .\|)$, $\hat{W}_\infty^{\mathcal{I}}(M, \lambda, p\|., .\|)$.

(vi) By a lacunary $\theta = (k_r)$; $r = 0, 1, 2, ...$ where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers with $k_r - k_{r-1}$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and $h_r = k_r - k_{r-1}$. As a final illustration let

$$b_{nk} = \begin{cases} \frac{1}{h_r}, & \text{if } k_{r-1} < k \leq k_r, \\ 0, & \text{otherwise} \end{cases}$$

Then we denote the above classes of sequences by $\hat{W}^{\mathcal{I}}(M, \theta, p\|., .\|)$, $\hat{W}_0^{I}(M, \theta, p\|., .\|)$, $\hat{W}_\infty(M, \theta, p\|., .\|)$, $\hat{W}_\infty^{\mathcal{I}}(M, \theta, p\|., .\|)$.

The following well-known inequality ([13], p.190) will be used in the study.

$$\text{If } 0 \leq p_k \leq \sup p_k = H,\ D = \max\left(1, 2^{H-1}\right)$$

then

$$|a_k + b_k|^{p_k} \leq D\left\{|a_k|^{p_k} + |b_k|^{p_k}\right\}$$

for all k and $a_k, b_k \in \mathbb{C}$. Also $|a|^{p_k} \leq \max(1, |a|^H)$ for all $a \in \mathbb{C}$.

Theorem 2.1. $\hat{W}^{\mathcal{I}}(B, M, p, \|., .\|)$, $\hat{W}_0^{\mathcal{I}}(B, M, p, \|., .\|)$, $\hat{W}_\infty^{\mathcal{I}}(B, M, p, \|., .\|)$ *are linear spaces.*

Proof. We will prove the assertion for $\hat{W}_0^{\mathcal{I}}(B, M, p, \|., .\|)$ only and the others can be proved similarly. Assume that $x, y \in \hat{W}_0^{I}(B, M, p, \|., .\|)$ and $\alpha, \beta \in \mathbb{R}$. In order to prove the result we need to find some ρ_3 such that

$$\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(\alpha x) +\ t_{km}(\beta x)}{\rho_3}, z\right\|\right)\right]^{p_k} \geq \varepsilon\right\} \in \mathcal{I} \text{ for some}$$

$\rho_3 > 0,$ uniformly in m,

Since $x, y \in \hat{W}_0^{\mathcal{I}}(B, M, p, \|, ., \|)$, there exist some positive ρ_1 and ρ_2 such that

$$\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_1}, z\right\|\right)\right]^{p_k} \geq \varepsilon\right\} \in \mathcal{I} \text{ for some } \rho_1 > 0, \text{ uniformly in } m.$$

$$\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_2}, z\right\|\right)\right]^{p_k} \geq \varepsilon\right\} \in \mathcal{I} \text{ for some } \rho_2 > 0, \text{ uniformly in } m.$$

Define $\rho_3 = max(2|\alpha|\rho_1, 2|\beta|\rho_2)$. Since M is non-decreasing and convex and also $\|., .\|$ is a $2-$norm, for each m,

$$\begin{aligned}
&\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}\left(\alpha x_k + \beta y_k\right)}{\rho_3}, z\right\|\right)\right]^{p_k} \\
&\leq \sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{\alpha t_{km}(x)}{\rho_3}, z\right\| + \left\|\frac{\beta t_{km}(x)}{\rho_3}, z\right\|\right)\right]^{p_k} \\
&\leq \sum_{k=1}^{\infty} b_{nk}\frac{1}{2^{p_k}}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_1}, z\right\| + \left\|\frac{t_{km}(x)}{\rho_2}, z\right\|\right)\right]^{p_k} \\
&\leq \sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_1}, z\right\| + \left\|\frac{t_{km}(x)}{\rho_2}, z\right\|\right)\right]^{p_k}
\end{aligned}$$

$$\leq D\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_1},z\right\|\right)\right]^{p_k}+D\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_2},z\right\|\right)\right]^{p_k}$$

where $D=\max(1,2^{H-1})$.

From the above inequality we get

$$\left\{n\in\mathbb{N}:\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}\left(\alpha x_k+\beta y_k\right)}{\rho_3},z\right\|\right)\right]^{p_k}\geq\varepsilon\right\}$$
$$\subseteq\quad\left\{n\in\mathbb{N}:D\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x)}{\rho_1},z\right\|\right)\right]^{p_k}\geq\frac{\varepsilon}{2}\right\}$$
$$\cup\left\{n\in\mathbb{N}:D\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}y_k}{\rho_2},z\right\|\right)\right]^{p_k}\geq\frac{\varepsilon}{2}\right\},\text{ uniformly in } m.$$

Two sets on the right hand side belong to $\mathcal{I}$ and this completes the proof. □

It is also easy to verify that the space $\hat{W}_{\infty}\left(B,M,p,\|.,.\|\right)$ is also a linear space and moreover we have

Theorem 2.2. *For any fixed $n\in\mathbb{N}$, $\hat{W}_{\infty}\left(B,M,p,\|.,.\|\right)$ is paranormed space with respect to the paranorm defined by*

$$g_n\left(x\right)=\inf_{z\in X}\left\{\rho^{\frac{p_n}{H}}:\left(\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x)}{\rho},z\right\|\right)\right]^{p_k}\right)^{\frac{1}{H}}\leq 1,\forall z\in X\right\},$$

uniformly in m.

Proof. The proof is parallel to the proof of the Theorem 2 in [20] and so is omitted. □

Theorem 2.3. *(i) Let $0<infp_k\leq p_k\leq 1$. Then $\hat{W}^{\mathcal{I}}\left(B,M,p,\|.,.\|\right)\subset\hat{W}^{I}\left(B,M,\|.,.\|\right)$.*
(ii) $1<p_k\leq\sup p_k\leq\infty$. Then $\hat{W}^{\mathcal{I}}\left(B,M,\|.,.\|\right)\subset\hat{W}^{\mathcal{I}}\left(B,M,p\,\|.,.\|\right)$.

Proof. (i) Let $(x_k)\in\hat{W}^{\mathcal{I}}\left(B,M,p,\|.,.\|\right)$. Since $0<infp_k\leq p_k\leq 1$, we have, for each m,

$$\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x-L)}{\rho},z\right\|\right)\right]\leq\sum_{k=1}^{\infty} b_{nk}\left[M\left(\left\|\frac{t_{km}(x-L)}{\rho},z\right\|\right)\right]^{p_k}$$

So
$$\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} b_{nk} \left[M\left(\left\|\frac{t_{km}(x-L)}{\rho}, z\right\|\right)\right] \geq \varepsilon\right\}$$
$$\subseteq \left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} b_{nk} \left[M\left(\left\|\frac{t_{km}(x-L)}{\rho}, z\right\|\right)\right]^{p_k} \geq \varepsilon\right\} \in \mathcal{I}, \text{ uniformly in } m.$$

(ii) Let $p_k \geq 1$ for each k, and $\sup p_k \leq \infty$. Let $(x_k) \in \hat{W}^I(B, M, p, \|.,.\|)$. Then for each $0 < \varepsilon < 1$ there exists a positive integer N such that
$$\sum_{k=1}^{\infty} b_{nk} \left[M\left(\left\|\frac{t_{km}(x-L)}{\rho}, z\right\|\right)\right] \leq \varepsilon < 1$$
for all $n \geq N$. This implies that
$$\sum_{k=1}^{\infty} b_{nk} \left[M\left(\left\|\frac{t_{km}(x-L)}{\rho_3}, z\right\|\right)\right]^{p_k} \leq \sum_{k=1}^{\infty} b_{nk} \left[M\left(\left\|\frac{t_{km}(x-L)}{\rho_3}, z\right\|\right)\right].$$
So we have
$$\left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} b_{nk} \left[M\left(\left\|\frac{t_{km}(x-L)}{\rho}, z\right\|\right)\right]^{p_k} \geq \varepsilon\right\}$$
$$\subseteq \left\{n \in \mathbb{N} : \sum_{k=1}^{\infty} b_{nk} \left[M\left(\left\|\frac{t_{km}(x-L)}{\rho}, z\right\|\right)\right] \geq \varepsilon\right\} \in \mathcal{I}, \text{ uniformly in } m.$$
This completes the proof. □

The following corollary follows immediately from the above theorem.

Corollary 2.1. *Let $B = (C, 1)$ Cesàro matrix and let M be an Orlicz function.*

(i) If $0 < \inf p_k \leq p_k < 1$, then $\hat{W}^I(M, p, \|.,.\|) \subset \hat{W}^I(M, \|.,.\|)$.

(ii) If $1 \leq p_k \leq \sup p_k < \infty$, then $\hat{W}^I(M, \|.,.\|) \subset \hat{W}^I(M, p\,\|.,.\|)$

Definition 2.1. *Let X be a sequence space. Then X is called solid if $(\alpha_k x_k) \in X$ whenever $(x_k) \in X$ for all sequences (α_k) of scalars with $|\alpha_k| \leq 1$ for all $k \in N$.*

Theorem 2.4. *The sequence spaces $\hat{W}_0^{\mathcal{I}}(B, M, p, \|,.,\|)$, $\hat{W}_\infty^{\mathcal{I}}(B, M, p, \|.,.\|)$ are solid.*

Proof. We give the proof for $\hat{W}_0^{\mathcal{I}}(B, M, p, \|.,.\|)$ only.

Let $(x_k) \in \hat{W}_0^{\mathcal{I}}(B, M, p, \|.,.\|)$ and let (α_k) be a sequence of scalars such that $|\alpha_k| \leq 1$ for all $k \in N$. Then we have

$$\left\{ n \in \mathbb{N} \ : \sum_{k=1}^{\infty} b_{nk} \left[M \left(\left\| \frac{t_{km}(\alpha_k x_k)}{\rho}, z \right\| \right) \right]^{p_k} \geq \varepsilon \right\}$$

$$\subseteq \left\{ n \in \mathbb{N} \ : C \sum_{k=1}^{\infty} b_{nk} \left[M \left(\left\| \frac{t_{km}(x)}{\rho}, z \right\| \right) \right]^{p_k} \geq \varepsilon \right\} \in \mathcal{I}, \text{ uniformly in } n,$$

where $C = \max_k \{1, |\alpha_k|^H\}$.

Hence $(\alpha_k x_k) \in \hat{W}_0^{\mathcal{I}}(B, M, p, \|.,.\|)$ for all sequences of scalars (α_k) with $|\alpha_k| \leq 1$ for all $k \in N$ whenever $(x_k) \in \hat{W}_0^{\mathcal{I}}(A, M, p, \|.,.\|)$. $\square$

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